Optimal Eventual Byzantine Agreement Protocols with Omission Failures

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Eventual Byzantine Agreement (EBA)

There are n agents $Agt = \{1, \ldots, n\}$, up to t of which are faulty.

Agents have initial preferences for which value (0 or 1) to decide on. They each can perform an action $\texttt{decide}_i(v)$.

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We require, for each value $v \in \{0, 1\}$,

- **Unique Decision:** all agents decide at most once
- **Agreement:** nonfaulty agents decide on the same value
- **Validity:** if a nonfaulty agent decides v, then some agent had initial preference v
- **Termination:** nonfaulty agents eventually decide

Communication Model

We focus on the *round-based, synchronous, message-passing* communication model with omission failures.

- **Sending-omission Failures:** A faulty agent may omit to send an arbitrary set of messages in any given round.

Designing Optimal Protocols

Dwork and Moses [1990][1] defined a notion of optimality for Byzantine agreement protocols.

P_1 **dominates protocol** P_2**:** if for all *corresponding* runs r_1 and r_2, for all agents j, we have $dt_1(r_1) \leq dt_2(r_2)$, where $dt_i(r_i)$ is the decision time of agent j running P_i in run r_i.

- A run r_1 of P_1 *corresponds* to a run r_2 of P_2 if r_1 and r_2 agree on all agents' inputs and the *failure pattern*
  - which agents are faulty and which of their messsages are sent

[1] C. Dwork and Y. Moses. 1990. Knowledge and common knowledge in a Byzantine environment: crash failures. *Information and Computation 88*, 2 (1990), 156–186.

Designing Optimal Protocols

Optimal protocols:

- For EBA, *optimal* protocols are the ones that are not dominated by any other protocol.
  - E.g., there is an optimal protocol where (roughly speaking) agents decide 0 as soon as possible, and one where agents decide 1 as soon as possible
    - neither dominates the other

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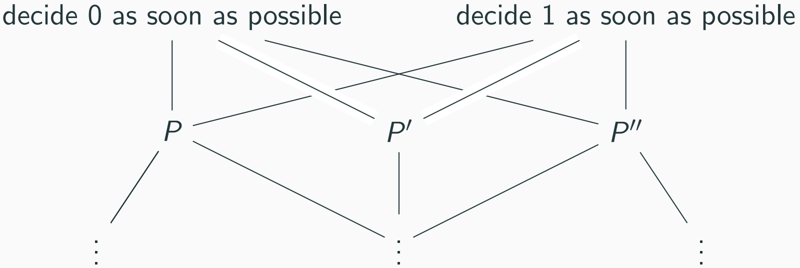

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But full information is costly, especially with limited bandwidth.

- What happens if we limit information exchange?

Our Main Idea

We can describe a protocol in terms of two separate components:

- the *information-exchange* protocol
    - determines what information is exchanged
- the *action* protocol,
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BA Protocol

action protocol P information-exchange protocol $\mathcal{E}$

A protocol P is optimal w.r.t. an information-exchange protocol $\mathcal{E}$ if P is optimal amongst protocols that use $\mathcal{E}$.

Our Results

- We provide a knowledge-based program $\mathbf{P}^0$ that gives an optimal protocol in two limited-information settings.

[2] J. Y. Halpern, Y. Moses, and O. Waarts. 2001. A characterization of eventual Byzantine agreement. SIAM J. Comput. 31, 3 (2001), 838–865. https://doi.org/10.1137/S0097539798340217

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- We provide a knowledge-based program $\mathbf{P}^0$ that gives an optimal protocol in two limited-information settings.
- We a provide a knowledge-based program $\mathbf{P}^1$ that generalizes $\mathbf{P}^0$ and gives an optimal protocol w.r.t. full-information exchange.

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- We a provide a knowledge-based program $\mathbf{P}^1$ that generalizes $\mathbf{P}^0$ and gives an optimal protocol w.r.t. full-information exchange.
- Moreover, $\mathbf{P}^1$ is implementable in polynomial time, which shows that a polynomial-time optimal protocol for EBA with omission failures exists, settling a question left open by Halpern, Moses, and Waarts[2] (HMW from now on).

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Reasoning about Knowledge

To model these systems semantically, we use the standard runs-and-systems model [Fagin et al., 1995].[3]

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An *interpreted system* is a pair $\mathcal{I} = (\mathcal{R}, \pi)$.

- $\mathcal{R}$ is the set of runs.
- π is an *interpretation function* that indicates which atomic propositions are true at each point (r, m) in the system.

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Formally, a run $r \in \mathcal{R}$ is a function mapping a time m to a *global state* $r(m)$:

- a tuple $(s_e, s_1, \ldots, s_n)$ describing the local state of the environment and the local state of each agent $i \in \{1, \ldots, n\}$.

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Reasoning about Knowledge

The semantics of the logic is given by $\mathcal{I}, (r, m) \models \phi$ where ϕ is a formula. Key definition:

- $(\mathcal{I}, (r, m)) \models K_i\phi$ if $(\mathcal{I}, (r', m)) \models \phi$ at all points (r', m) such that i has the same local state in (r, m) and (r', m)

Another useful formula for EBA:

- $(\mathcal{I}, (r, m)) \models \exists 0$ if some agent's initial state in r is 0.

Knowledge-based programs

Knowledge-based programs [Halpern and Fagin, 1985][4] are high-level abstractions.

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- *Standard programs* are ones where the tests directly depend on the agent i's local state.
- For knowledge-based programs, the tests can be Boolean combinations of $K_i\psi$

$$\text{if } K_i(\exists 0) \text{ then } \texttt{decide}_i(0) \textit{ else } \ldots$$

A standard program P *implements* a knowledge-based program $\mathbf{P}$ in an intepreted system $\mathcal{I}$ if the knowledge-based conditions in $\mathbf{P}$ hold at points in $\mathcal{I}$ exactly when the standard conditions in P hold.

$$\text{if } \mathit{init}_i = 0 \wedge \text{received}_i(0) \text{ then } \texttt{decide}_i(0) \textit{ else } \ldots$$

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Crash $\rightarrow$ omission failures

One idea is to decide 0 agressively [HMW, Castañeda et al., 2014][5]

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Consider a protocol that decides 0 when $K_i(\exists 0)$ holds.

- This works for crash failures, but with omission failures we run into a problem.
- Agent i can't always decide 0 when $K_i(\exists 0)$ holds when we have omission failures.

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Knowledge-based program $\mathbf{P}^0$

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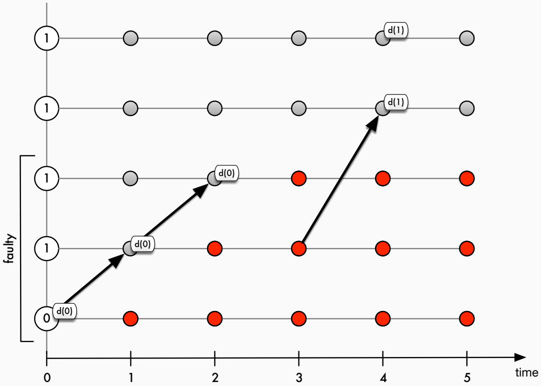

Knowledge-based program P^0

- Agent *i* decides 0 if *i* knows that someone *just decided* 0 or *i* has initial preference 0.
- Agent *i* decides 1 if *i* knows that no agent is currently deciding 0.

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Program: $\mathbf{P}_i^0$

if $\mathit{decided}_i \neq \bot$ **then** `noop`
else if $\mathit{init}_i = 0 \ \vee \ K_i(\bigvee_{j \in Agt} \mathit{jdecided}_j = 0)$ **then** $\texttt{decide}_i(0)$
else if $K_i(\bigwedge_{j \in Agt} \neg(\mathit{deciding}_j = 0))$ **then** $\texttt{decide}_i(1)$
else `noop`

This represents the *action protocol*. We will investigate it with respect to different information-exchange protocols.

Optimality of $\mathbf{P}^0$

In the paper, we give a sufficient condition for the optimality of $\mathbf{P}^0$ with respect to different information exchanges.

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We then define two limited-information settings, $\mathcal{E}_{\min}$ and $\mathcal{E}_{basic}$, that satisfy this condition.

P^0 is optimal in two limited-information settings

In $\mathcal{E}_{\mathrm{min}}$, agents keep track only of the time, their initial value, whether they have decided, and whether they received a message from another agent.

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- **Local states:**
  - $time_i$: records the time
  - $init_i$: records the initial preference of the agent
  - $decided_i$: records the agent's decision
  - jd_i: records whether agent i received a message from an agent that decided.

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  - $decided_i$: records the agent's decision
  - jd_i: records whether agent i received a message from an agent that decided.
- **Messages:** 0 or 1. Each agent send a message once, only when they decide.

Now we consider the action protocol that implements $\mathbf{P}^0$ with respect to this information exchange.

Implementing $\mathbf{P}^0$ in $\mathcal{E}_{\mathsf{min}}$

The following standard program implements $\mathbf{P}^0$ in $\mathcal{E}_{\mathsf{min}}$:

- agent i decides 0 if it has an initial value of 0 or it received the message 0 in the previous round.
- agent i decides 1 in round $t+1$ if it hasn't decided 0

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Program: P_i^{min}

if $decided_i \neq \bot$ **then** `noop`
else if $init_i = 0 \vee jd_i = 0$ **then** $\texttt{decide}_i(0)$
else if $time_i = t+1$ **then** $\texttt{decide}_i(1)$
else `noop`

This is just the usual EBA program; it is optimal with limited-information exchange!

P^0 is optimal in two limited-information settings

In $\mathcal{E}_{basic}$, agents keep track of everything in $\mathcal{E}_{\min}$ + how many messages of the form $(init, 1)$ (which encodes "I haven't heard about a 0") they received in the last round.

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In $\mathcal{E}_{basic}$, agents keep track of everything in $\mathcal{E}_{\min}$ + how many messages of the form $(init, 1)$ (which encodes "I haven't heard about a 0") they received in the last round.

- **Local states:** same as in $\mathcal{E}_{min}$ except one additional variable,
  - $\#1_i$ counts the number of messages of the form $(init, 1)$ that i receives in the previous round.
- **Messages:** 0, 1 or $(init, 1)$.
  - 0 (resp., 1) is sent when the agent decides 0 (resp., 1);
  - $(init, 1)$ is sent if the agent hasn't decided yet.

Now, the action protocol that implements $\mathbf{P}^0$ can do better compared to the action protocol that implements $\mathbf{P}^0$ in the minimal information-exchange.

Implementing $\mathbf{P}^0$ in $\mathcal{E}_{basic}$

The following standard program implements $\mathbf{P}^0$ in $\mathcal{E}_{basic}$:

Now agent i can decide if it hears from enough agents that they haven't decided 0.

- This is guaranteed to happen by round $t + 1$, but may happen earlier

Implementing $\mathbf{P}^0$ in $\mathcal{E}_{basic}$

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Now agent i can decide if it hears from enough agents that they haven't decided 0.

- This is guaranteed to happen by round $t + 1$, but may happen earlier

Program: P_i^{basic}

if $\mathit{decided}_i \neq \bot$ **then** `noop`
else if $\mathit{init}_i = 0 \vee \mathit{jd}_i = 0$ **then** $\texttt{decide}_i(0)$
else if $\#1_i > n - \mathit{time}_i \vee \mathit{jd}_i = 1$ **then** $\texttt{decide}_i(1)$
else `noop`

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- It can be shown that this suffices for the nonfaulty agents to decide 1 in round 3.
- However, with $\mathbf{P}^0$, they don't decide until round 11
  - no agent knows that there is no chain of 0s until round 11.

Common knowledge

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- Every nonfaulty agent knows φ: $E_{\mathcal{N}}\varphi$
- φ is common knowledge among the nonfaulty agents: $C_{\mathcal{N}}\varphi$
    - All the nonfaulty agents know φ, all the nonfaulty agents know that all the nonfaulty agents know φ, ...

$$C_{\mathcal{N}}\varphi \quad \Leftrightarrow \quad E_{\mathcal{N}}\varphi \wedge E_{\mathcal{N}}E_{\mathcal{N}}\varphi \wedge E_{\mathcal{N}}E_{\mathcal{N}}E_{\mathcal{N}}\varphi \wedge \cdots$$

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We can modify $\mathbf{P}^0$ to get $\mathbf{P}^1$, which makes use of this:

Program: $\mathbf{P}_i^1$

if $\textit{decided}_i \neq \bot$ **then** `noop`
else if $K_i(C_{\mathcal{N}}(\textit{t-faulty} \wedge \textit{no-decided}_{\mathcal{N}}(1) \wedge \exists 0)$ **then** $\texttt{decide}_i(0)$
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Showing $\mathbf{P}^1$ is optimal

In the paper, we show that

- $\mathbf{P}^1$ is equivalent to $\mathbf{P}^0$ in the limited-exchange settings we considered earlier
- $\mathbf{P}^1$ is also optimal in the full-information exchange setting.
    - The proof makes use of the *continual common knowledge* operator $C_{\mathcal{S}}^{\boxdot}\phi$ introduced by HMW. It is defined analogously to common knowledge except that $E_{\mathcal{S}}\phi$ is replaced by $\boxdot E_{\mathcal{S}}\phi$.
        - HMW provides a characterization of optimality w.r.t. full-information exchange in terms of continual common knowledge.

Implementing P[1]

The continual common knowledge characterization of optimality w.r.t. full-information introduced by HMW cannot be implemented efficiently in general.

[6] Y. Moses and M. R. Tuttle. 1988. Programming simultaneous actions using common knowledge. *Algorithmica 3* (1988), 121–169. https://doi.org/10.1007/BF01762112.

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The continual common knowledge characterization of optimality w.r.t. full-information introduced by HMW cannot be implemented efficiently in general.

- But $\mathbf{P}^1$ uses only common knowledge, not continual common knowledge
- $\mathbf{P}^1$ does have a polynomial-time implementation P^{fip}, which uses the compact communication-graph representation of [Moses and Tuttle, 1988].[6]

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This shows that there exists polynomial-time protocols for EBA that are optimal (in general).

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Limited-information vs. full-information exchange

Total number of bits communicated:

- $P^{\min}$: n^2
- P^{basic}: $O(n^2 t)$
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Decision times in a failure-free run:

- If $\exists 0$ in the run, then all agents decide by round 2 in $P^{\min}, P^{basic}, P^{fip}$.
- If $\neg\exists 0$ in the run, then all agents decide by:
  - round $t + 2$ in $P^{\min}$
  - round 2 in P^{basic}, P^{fip}

Given the tradeoffs, P^{basic} might be preferable to P^{fip}.

Further work

- Characterizing optimality with respect to an information-exchange $\mathcal{E}$.
- Finding other optimal protocols for EBA under omission failures.
- Investigating optimality w.r.t. limited-information exchange in other settings.